AP Pre-Calculus Name ____

Prerequisite Assignment

AP PreCalculus centers on functions modeling dynamic phenomena. The course fosters the development of a deep conceptual understanding of functions.

The Prerequisite Assignment:

Students need a strong foundation to be ready for the rigorous work required throughout the course. Completing the prerequisite assignment should help review topics (see list below) studied in Algebra, Geometry and Advanced Algebra.

- Proficiency with the skills and concepts related to linear and quadratic functions, including algebraic manipulation, solving equations, and solving inequalities.
- Proficiency in manipulating algebraic expressions related to polynomial functions, including polynomial addition and multiplication, factoring quadratic trinomials, and using the quadratic formula.
- Proficiency in solving right triangle problems involving trigonometry.
- Proficiency in solving systems of equations in two and three variables.
- Familiarity with piecewise-defined functions.
- Familiarity with exponential functions and rules for exponents.
- Familiarity with radicals (e.g. square roots, cube roots).
- Familiarity with complex numbers.
- Familiarity with communicating and reasoning among graphical, numerical, analytical, and verbal representations of functions.

Directions for Prerequisite Assignment:

1) Use the following link to watch the pre-requisite videos prepared by College Board to help review topics: https://www.youtube.com/playlist?list=PLoGgviqq48477akYFkAyTSeTbWkF4hoWk

2) You need to show ALL work with logical steps. DO NOT list only an answer. Work neatly and in an organized fashion. Please expect a test within the first few days of the school year over this material. You may ask questions over the packet and any other pre-requisite information when we return in August. We will begin new material by the 4th day of school, so it is imperative that these skills are mastered in order to be successful moving forward in the course.

Calculators:

Students enrolled in AP PreCalculus will be using a graphing calculator throughout the course. A graphing calculator is required on the AP test. A list of acceptable calculators for the AP test is available online at:

https://apcentral.collegeboard.org/exam-administration-ordering-scores/administering-exams/on-exam-day/calculator-policy#list

AP Pre-Calculus
Summer Prep Packet

PART I: LINEAR FUNCTIONS

Write the standard form equation of the line through the given points.

1) (0, 2) and (-4, 4)

2) (0,1) and (-1, -3)

Write the slope-intercept form equation of the line described below .

3) through: (4,1), parallel to $y = -\frac{1}{4}x + 4$

Write the standard form equation of the line described below.

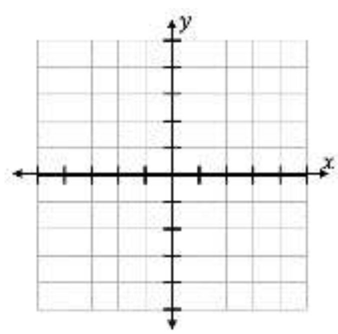
4) through: (1, -4), perpendicular to $y = \frac{1}{8}x + 4$

Write the point-slope form equation of the line through the given points.

5) (0, 4) and (-3, 3)

Find the x- and y- intercepts. Then use them to graph the line on the provided grid.

6) $7x - 3y = 6$

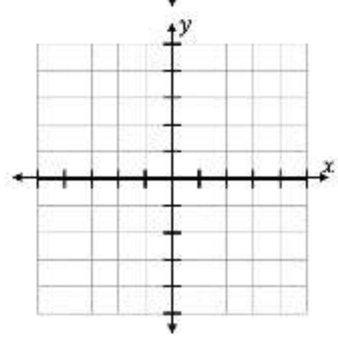


Graph the following lines on the provided grid.

7) $x = -3$

8) $y = 4$

9) $y = 3x$

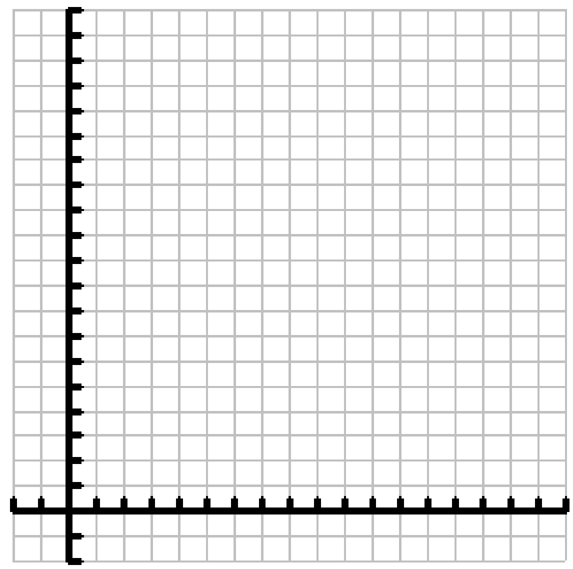


Write an equation to model the given situation. Define all variables. Determine the slope and its meaning. Determine the y-intercept and its meaning. Sketch the graph with clearly labeled axes.

10) Shayla takes a cab from her office to the airport. The cab company charges \$0.75 per mile plus a \$4.00 flat convenience fee. Write an equation to model the total charge for the cab ride.

NOTE: USE DECIMALS IF NECESSARY; ROUND THEM TO NEAREST HUNDREDTH.

a. Variables:



b. Equation:

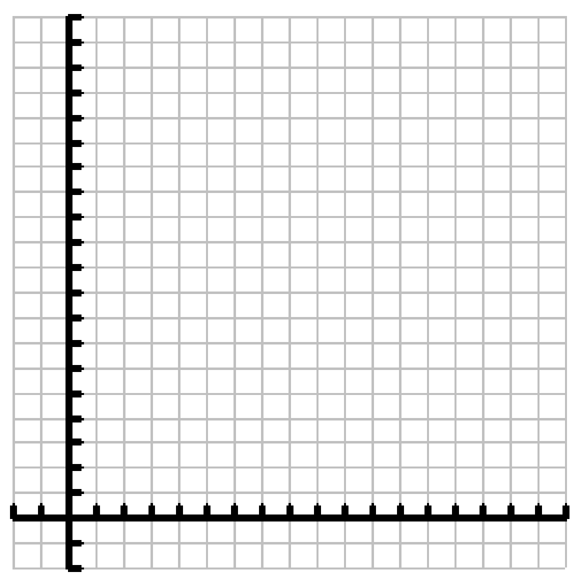
c. Slope: Meaning:

d. Y-intercept: Meaning:

11) Jimmy runs a company that is unfortunately laying off workers because of the economy. At the beginning of the year, the company has 90 employees, but with consistent layoffs, there are 78 employees left 4 months later. Write an equation to model the number of employees at Jimmy's company at a particular time.

NOTE: USE DECIMALS IF NECESSARY; ROUND THEM TO NEAREST HUNDREDTH.

a. Variables:



b. Equation:

c. Slope: Meaning:

d. Y-intercept: Meaning:

PART II: SOLVING EQUATIONS AND INEQUALITIES

Solve each equation or inequality.

1) $5(m+5)+3=3(2m+5)$

2) $-\frac{77}{12}+\frac{7}{2}a=-\frac{1}{2}\left(\frac{10}{3}a+\frac{7}{6}\right)+\frac{5}{8}$

3) $\frac{37}{8}\left(\frac{3}{2}n+\frac{1}{3}\right)=-\frac{77}{96}+\frac{9}{4}n$

4) $-8(1+m)-6m\geq-106$

5) $1\leq 7+2n<15$

6) $-\frac{401}{20}-8r<\frac{11}{8}\left(\frac{11}{2}r-1\right)$

PART III: POLYNOMIALS: ADDITION, SUBTRACTION, AND MULTIPLICATION

Simplify the following expressions.

1) $(14x^4-3x^2+2)+(3x^3+4x^2+5)$

2) $(5-x^4-2x^3)-(-6x^2+5x+5)$

3) $(8m^2-1)(3m^2-4m+5)$

4) $(4x-3y)^2$

Divide the expressions using long division.

5) $(2x^3 - 7x^2 - 22x + 20) \div (2x + 5)$

6) $(4x^4 + 3x^3 + 2x + 1) \div (x^2 + x + 2)$

Divide the expressions using synthetic division.

7) $(3x^3 + 5x^2 + 15) \div (x - 8)$

8) $(3x^3 - 6x + 7) \div (x - 2)$

PART IV: FACTORING QUADRATIC TRINOMIALS

Factor each expression completely.

1) $3x^2 - 108$

2) $3x^2 + 24x$

3) $4x^2 - 8x + 4$

4) $-5x^2 + 51x - 54$

5) $15x^3 - 70x^2 - 120x$

6) $-9x^3 + 18x^2 + 15x - 30$

7) $-12x^3 - 18x^2$

8) $x^3y^6 - 64$

PART V: SOLVING QUADRATIC EQUATIONS AND INEQUALITIES

Find the roots of each equation using the quadratic formula.

1) $3x^2 - 5 = 2x$

2) $10x^2 = -9$

Solve the following quadratic word problems ALGEBRAICALLY.

3) A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?

4) The sum of two positive numbers is 14 and their product is 32. What are the numbers?

5) An object is launched straight up into the air with an initial velocity of 64 ft/sec. It is launched from a height of 6 feet off the ground. Its height H, in feet, at t seconds is given by the equation $H = -16t^2 + 64t + 6$. Find all times t that the object is at height of 54 feet off the ground.

6) A clown in a circus is launched at 12.7 meters per second (m/s) from a 39.2-meter tall platform. A function for the clown's height at time t seconds after launch is $S(t) = -4.9t^2 + 12.7t + 39.2$ where s is in meters. When does the clown strike the ground?

7) The base of a triangle is 1 centimeter shorter than the altitude. If the area is 15 cm², what is the length of the altitude?

Solve each equation using factoring.

8) $7x^2 - 3 = -4x$

9) $4x^2 - 13x - 31 = 4$

10) $-10 = -13x - 5x^2 - 4$

11) $16x^3 - 12x^2 - 40x = 0$

Solve the following quadratic inequalities ALGEBRICALLY. State the solution set in interval notation. Draw a number line and shade it accordingly. If required, state the solution in DECIMAL FORM, 2 DECIMAL PLACES.

12) $0 < -2x^2 + 8x - 5$

13) $x^2 - 4x + 4 \leq 0$

PART VI: QUADRATIC FUNCTIONS

Write the quadratic equation in vertex form given the following information.

1) Vertex at (-2, 10) & passes through (-6, -12)

2) Minimum at (-1, -10) & y-intercept at (0, -9)

Write the following equations in vertex form by completing the square. Then state the vertex and whether it is a minimum or maximum.

3) $x^2 + 5x + 18 = y$

4) $x^2 - x = -7$

5) $f(x) = 5x^2 - 20x - 11$

6) $-4x^2 - 56x + 19 = 10 + y$

Determine the type of model that best fits the data (quadratic, linear or exponential) and calculate the average rate of change of the function on the given interval.

7)

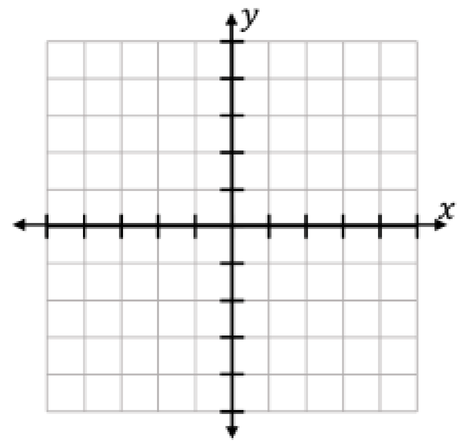
x	$f(x)$
-2	1/4
-1	1/2
0	1
1	2
2	4

Interval: $[-2, 1]$

8) (-3, -2), (-2, -3), (-1, -1), (0, 4), (1, 12)
Interval: $[-3, 0]$

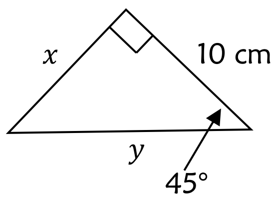
Find the average rate of change of the given function on the given interval. Sketch a graph of the parabola with the secant line that represents the average rate of change on the interval.

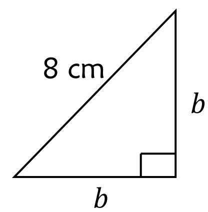
9) $f(x) = -2(x+5)^2 + 3$
Interval: $[-5, -4]$

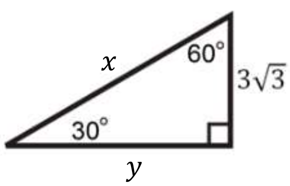


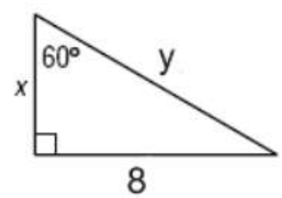
PART VII: RIGHT TRIANGLE TRIGONOMETRY

Find the value of all variables below using special right triangle ratios. Do not leave radicals in the denominators of your fractions.

1)


2)


3)


4)


Use a trigonometric ratio to solve the following problems.

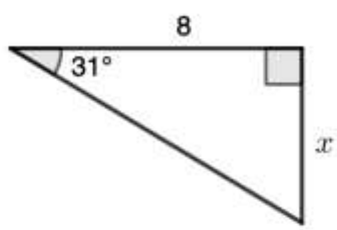
5) An ant is looking up at you with an angle of elevation of 48°. You are 6 feet tall. How far is the ant from your foot? Round to 2 decimal places.

6) A baseball diamond is in the shape of a square with each side being 90 feet. If the catcher throws out a runner at second base who was trying to steal, how far does he need to throw the ball?

7) The shorter leg of a $30° - 60° - 90°$ triangle is 7.4 meters long. Find the perimeter.

8) Find the altitude of an equilateral triangle if each side of the triangle has a length of 14 meters.

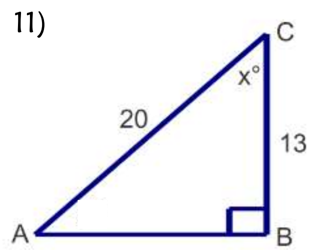
9) Solve for *x*.



Find the value of the missing angle.

10) If $\sin\theta = \frac{3}{7}$, find θ.

11)



PART VIII: SOLVING SYSTEMS OF EQUATIONS

Solve the system of equations algebraically.

1) $25x - 5y = 30$
$-35x + 7y = -7$

2) $8x - y + 3z = -38$
$2x + 5y - 4z = 32$
$x - y + z = -9$

3) $5x - 2y = 14$
$-3x + y = -7$

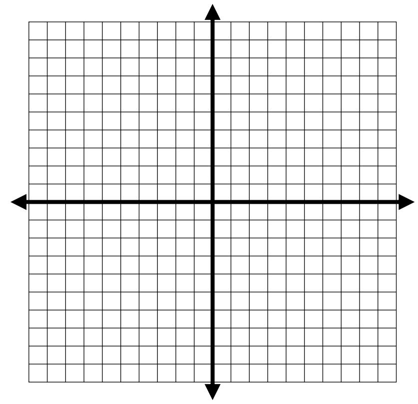
4) $-4x - 2y - z = 5$
$2x - z = 8$
$y - 2z = -4$

PART IX: PIECEWISE FUNCTIONS

Graph the following functions. Then write the absolute value function as a piecewise function.

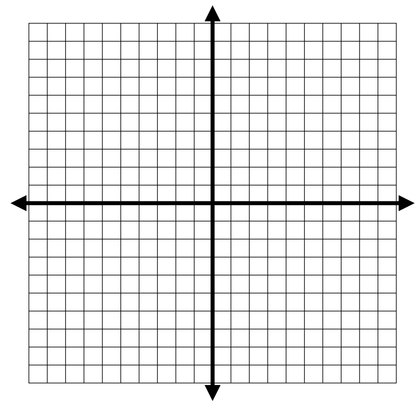
1) $f(x) = |x^2 - 1|$

Piecewise function:



2) $f(x) = |(x-1)^2 - 9|$

Piecewise function:



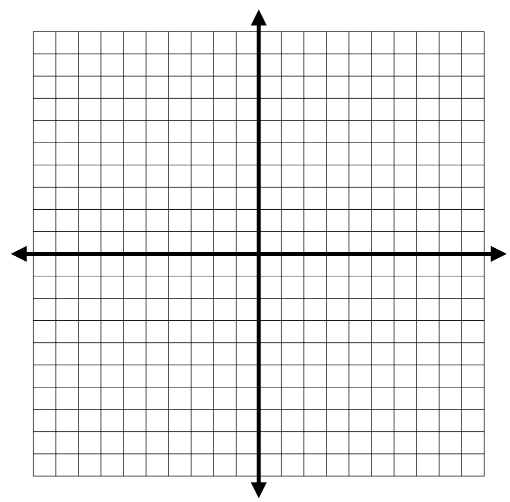
Graph the following functions. Then evaluate them at requested values.

3) $h(x) = \begin{Bmatrix} -e^x + 7, -\infty < x < 0 \\ 3, 0 < x < 3 \\ |2x+2|, 3 \le x < 4 \end{Bmatrix}$

a. $h(-10) =$

b. $h(0) =$

c. $h(2) =$

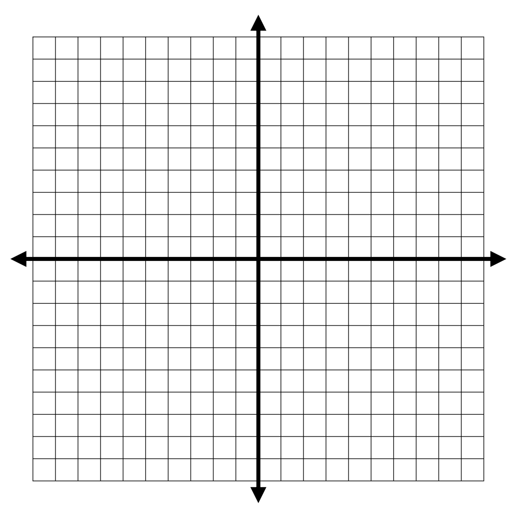


4) $k(x) = \begin{Bmatrix} 3 - x, x < -2 \\ \frac{4}{x-1}, -2 < x < 3 \\ (x-4)^3, 3 \le x < 6 \end{Bmatrix}$

a. $k(-1) =$

b. $k(1) =$

c. $k(5) =$



PART X: EXPONENTIAL FUNCTIONS

1) The population of a species of ant grows at a rate of 13% every six months. If there are initially 85 ants in the colony, how many would be present after 3.5 years?

2) The half-life of a radioactive isotope is 10 years. Initially, there are 500 grams of the isotope in the laboratory. How much of the isotope would remain after 33 years?

3) The value of a car depreciates by 17% a year. If a car is initially purchased for $28,000, what would be the worth of the car after 5 years?

4) Andi invests $7,500 in a savings account that pays 2.55% interest compounded monthly. How much money will be in the account after 20 years?

5) The population of Hanover was 38,721 in the year 1990. The population has grown consistently at a rate of 1.6% each year. What was the population in the year 2010 (the last census) and what would be the predicted population for the year 2014?

6) The population of Hancock County, GA has declined steadily at a rate of 0.9% each year. In the year 1990, the population was approximately 8,750. What was the population in the year 2010, and would be the expected population for the year 2016?

PART XI: RULES OF EXPONENTS

Use properties of exponents to simplify the expressions completely.

1) $b^4 \cdot b^2$

2) $x^{-3} \cdot x^5$

3) $\frac{m^2}{m^6}$

4) $\left(\frac{x}{x^2y}\right)^2$

5) $(5y)^2$

6) $\left(\frac{z^9}{3z^5}\right)^{-1}$

7) $\left(\frac{n}{4m^2n}\right)^{-2}$

8) $\frac{2^{-4}2^2}{(2^2)^{-4}}$

9) $\frac{\left(2(2^2)^{-3}\right)^2}{2^{-5}}$

10) $\frac{\left(-x^{-1}y^2\right)^{-1}}{x^{-2}y^3(x^0y^{-2})^3}$

11) $\frac{x^{-3}y^3}{-x^4y^5(-y^{-3})^2}$

PART XII: SIMPLIFYING RADICALS

Simplify the following expressions as much as possible.

1) $\sqrt{75x^7y^5}$

2) $\sqrt{27a^{11}b^7}$

3) $\sqrt{32a^7b^4}$

4) $\frac{4}{\sqrt{8}}$

5) $\frac{5}{\sqrt{125}}$

6) $-\frac{4}{3\sqrt{2}}$

7) $4\sqrt{5}+\sqrt{125}+\sqrt{45}$

8) $\sqrt{48}+10-\sqrt{100}-\sqrt{98}$

9) $2x\sqrt{3x^2}+5-7\sqrt{3x^2}-2$

10) $3x\sqrt{18}-3\sqrt{98x^2}+2x\sqrt{12x^2}$

PART XIII: COMPLEX NUMBERS

Simplify the following expressions as much as possible.

1) $(3i^{31}-3)(4+i^{57})$

2) $(i\sqrt{2}-6)-(3i\sqrt{32}+15)+(-2i\sqrt{18}-11)$

3) $i\sqrt{3}\left(-2-i\sqrt{6}\right)$

4) $\frac{3-4i}{3i}$

AP Pre-Calculus
Summer Prep Packet

Name Key

PART I: LINEAR FUNCTIONS → ($Ax+By=C$)

Write the standard form equation of the line through the given points.

1) (0, 2) and (-4, 4)

$m=\frac{4-2}{-4-0}=\frac{2}{-4}=\frac{-1}{2}$

$y-2=-\frac{1}{2}(x-0)$

$y=-\frac{1}{2}x+2$ → $(\frac{1}{2}x+y=2)\cdot 2$

$\boxed{x+2y=4}$

2) (0,1) and (-1, -3)

$m=\frac{-3-1}{-1-0}=\frac{-4}{-1}=4$

$y-1=4(x-0)$

$y-1=4x$

$y=4x+1$

$(-4x+y=1)\cdot -1$

$\boxed{4x-y=-1}$

Write the slope-intercept form equation of the line described below. ($y=mx+b$)

3) through: (4,1), parallel to $y=-\frac{1}{4}x+4$

$m=-\frac{1}{4}$

(4,1)

$y-1=-\frac{1}{4}(x-4)$

$y-1=-\frac{1}{4}x+1$

$+1 \qquad +1$

$\boxed{y=-\frac{1}{4}x+2}$

Write the standard form equation of the line described below.

4) through: (1, -4), perpendicular to $y=\frac{1}{8}x+4$

$m=-8$

(1,-4)

$y+4=-8(x-1)$

$y+4=-8x+8$

$y=-8x+4$

$\boxed{8x+y=4}$

Write the point-slope form equation of the line through the given points. → ($y-y_1=m(x-x_1)$)

5) (0, 4) and (-3, 3)

$m=\frac{3-4}{-3-0}=\frac{-1}{-3}=\frac{1}{3}$

$\boxed{y-3=\frac{1}{3}(x+3)}$

Find the x- and y- intercepts. Then use them to graph the line on the provided grid.

6) $7x-3y=6$

x-int: $7x-3(0)=6$

$7x=6$

$x=\frac{6}{7}$

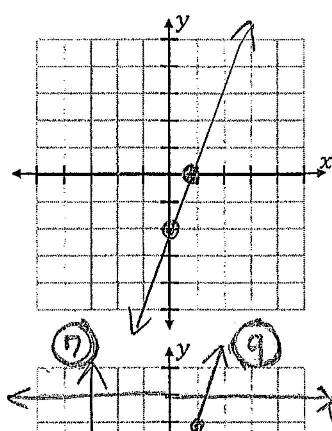
$\boxed{(\frac{6}{7},0)}$

y-int: $7(0)-3y=6$

$-3y=6$

$y=-2$

$\boxed{(0,-2)}$



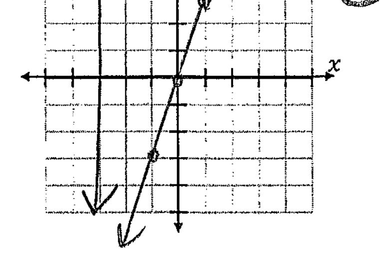
Graph the following lines on the provided grid.

7) $x=-3$

8) $y=4$

9) $y=3x$

(see labeled graph) ⟶



Write an equation to model the given situation. Define all variables. Determine the slope and its meaning. Determine the y-intercept and its meaning. Sketch the graph with clearly labeled axes.

10) Shayla takes a cab from her office to the airport. The cab company charges $0.75 per mile plus a $4.00 flat convenience fee. Write an equation to model the total charge for the cab ride.

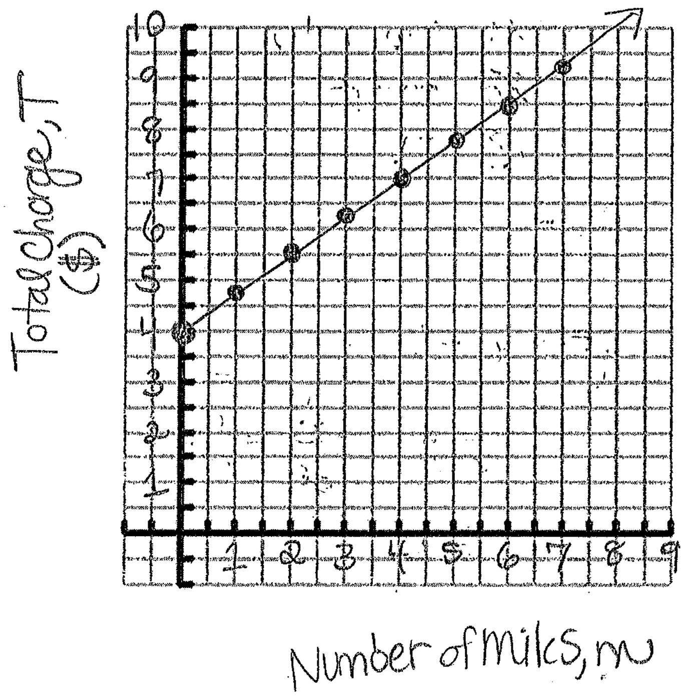
NOTE: USE DECIMALS IF NECESSARY; ROUND THEM TO NEAREST HUNDREDTH.

a. Variables: T = total charge
m = number of miles in cab

b. Equation:
$T = 0.75m + 4$

c. Slope: 0.75 Meaning: the charge/mile

d. Y-intercept: (0,4) Meaning: the flat convenience fee



11) Jimmy runs a company that is unfortunately laying off workers because of the economy. At the beginning of the year, the company has 90 employees, but with consistent layoffs, there are 78 employees left 4 months later. Write an equation to model the number of employees at Jimmy's company at a particular time.

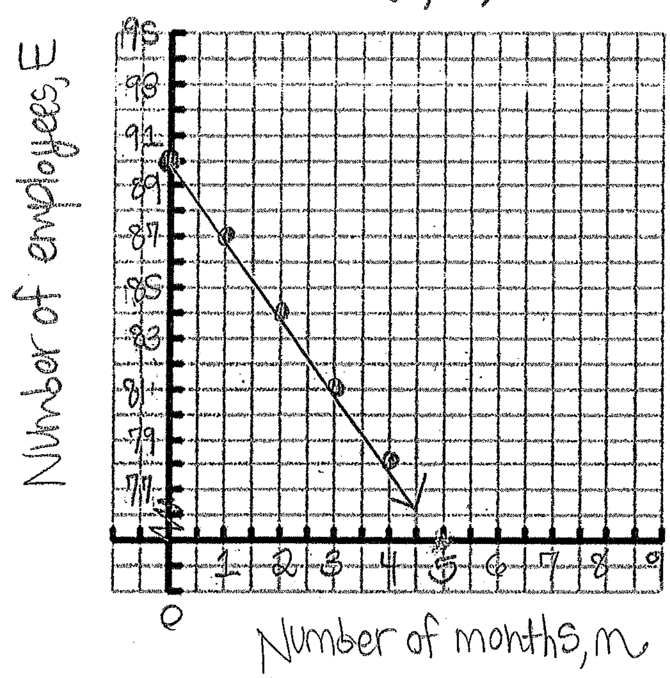
NOTE: USE DECIMALS IF NECESSARY; ROUND THEM TO NEAREST HUNDREDTH. (0,90) (4,78) $\frac{78-90}{4-0} = \frac{-12}{4}$

a. Variables: E = number of employees
m = number of months

b. Equation:
$E = -3m + 90$

c. Slope: -3 Meaning: company is losing 3 employees/month

d. Y-intercept: (0,90) Meaning: Starting amount of employees at company before layoffs



PART II: SOLVING EQUATIONS AND INEQUALITIES

Solve each equation or inequality.

1) $5(m+5)+3=3(2m+5)$

$5m+25+3=6m+15$

$5m+28=6m+15$

$28=m+15$

$13=m$

2) $-\frac{77}{12}+\frac{7}{2}a=-\frac{1}{2}\left(\frac{10}{3}a+\frac{7}{6}\right)+\frac{5}{8}$

$-\frac{77}{12}+\frac{7}{2}a=-\frac{10}{6}a-\frac{7}{12}+\frac{5}{8}$

$-\frac{77}{12}+\frac{7}{2}a=-\frac{10}{6}a+\frac{1}{24}$

$-\frac{77}{12}=-\frac{31}{6}a+\frac{1}{24}$

$-\frac{155}{24}=-\frac{31}{6}a$, $a=\frac{5}{4}$

3) $\frac{37}{8}\left(\frac{3}{2}n+\frac{1}{3}\right)=-\frac{77}{96}+\frac{9}{4}n$

$\frac{111}{16}n+\frac{37}{24}=\frac{-77}{96}+\frac{9}{4}n$

$\frac{111}{16}n=\frac{-75}{32}+\frac{9}{4}n$

$\frac{75}{16}n=\frac{-75}{32}$

$n=-\frac{1}{2}$

4) $-8(1+m)-6m\geq-106$

$-8-8m-6m\geq-106$

$-8-14m\geq-106$

$-14m\geq-98$

$m\leq7$

5) $1\leq7+2n<15$

$-7\quad-7\quad\quad-7$

$\frac{-6}{2}\leq\frac{2n}{2}<\frac{8}{2}$

$-3\leq n<4$

6) $-\frac{401}{20}-8r<\frac{11}{8}\left(\frac{11}{2}r-1\right)$

$-\frac{401}{20}-8r<\frac{121}{16}r-\frac{11}{8}$

$-8r<\frac{121}{16}r+\frac{747}{40}$

$\frac{-249}{16}r<\frac{747}{40}$

$r>-\frac{6}{5}$

PART III: POLYNOMIALS: ADDITION, SUBTRACTION, AND MULTIPLICATION

Simplify the following expressions.

1) $(14x^4-3x^2+2)+(3x^3+4x^2+5)$

$14x^4+3x^3+x^2+7$

2) $(5-x^4-2x^3)-(-6x^2+5x+5)$

$5-x^4-2x^3+6x^2-5x-5$

$-x^4-2x^3+6x^2-5x$

3) $(8m^2-1)(3m^2-4m+5)$

$24m^4-32m^3+40m^2-3m^2+4m-5$

$24m^4-32m^3+37m^2+4m-5$

4) $(4x-3y)^2$

$(4x-3y)(4x-3y)$

$16x^2-12xy-12xy+9y^2$

$16x^2-24xy+9y^2$

Divide the expressions using long division.

5) $(2x^3 - 7x^2 - 22x + 20) \div (2x + 5)$

$x^2 - 6x + 4$

$2x+5 \,/\, 2x^3 - 7x^2 - 22x + 20$

$-(2x^3 + 5x^2)$

$-12x^2 - 22x$

$-(-12x^2 - 30x)$

$8x + 20$

$-(8x + 20)$

0

$x^2 - 6x + 4$

no remainder

6) $(4x^4 + 3x^3 + 2x + 1) \div (x^2 + x + 2)$

$4x^2 - x - 7$

$x^2+x+2 \,/\, 4x^4 + 3x^3 + 0x^2 + 2x + 1$

$-(4x^4 + 4x^3 + 8x^2)$

$-x^3 - 8x^2 + 2x$

$-(-x^3 - x^2 - 2x)$

$-7x^2 + 4x + 1$

$-(-7x^2 - 7x - 14)$

$11x + 15$

$4x^2 - x - 7 + \frac{11x+15}{x^2+x+2}$

Divide the expressions using synthetic division.

7) $(3x^3 + 5x^2 + 15) \div (x - 8)$

$0x$ $\quad x = 8$

8	3	5	0	15
	↓	24	232	1856
	3	29	232	1871

$\left(3x^2 + 29x + 232 + \frac{1871}{x-8}\right)$

8) $(3x^3 - 6x + 7) \div (x - 2)$

$0x^2$ $\quad x = 2$

2	3	0	-6	7
	↓	6	12	12
	3	6	6	19

$3x^2 + 6x + 6 + \frac{19}{x-2}$

PART IV: FACTORING QUADRATIC TRINOMIALS

Factor each expression completely.

1) $3x^2 - 108$ (difference of squares)

$3(x^2 - 36)$

$3(x+6)(x-6)$

2) $3x^2 + 24x$

$3x(x+8)$

3) $4x^2 - 8x + 4$

$4(x^2-2x+1)$

$4(x-1)(x-1)$

4) $-5x^2 + 51x - 54$

$-(5x^2-51x+54)$

$-(5x-45)(5x-6)$

$-(x-9)(5x-6)$

5) $15x^3 - 70x^2 - 120x$

$5x(3x^2-14x-24)$

$5x(3x+4)(x-6)$

6) $-9x^3 + 18x^2 + 15x - 30$

$-(9x^3-18x^2-15x+30)$

$-(9x^2(x-2)-15(x-2))$

$-(x-2)(9x^2-15)$

7) $-12x^3 - 18x^2$

$-6x^2(2x+3)$

8) $x^3y^6 - 64$ (difference of cubes)

$(xy^2-4)(x^2y^4+4xy^2+16)$

PART V: SOLVING QUADRATIC EQUATIONS AND INEQUALITIES

Find the roots of each equation using the quadratic formula.

1) $3x^2 - 5 = 2x$

$3x^2-2x-5=0$

$b^2-4ac = (-2)^2-4(3)(-5) = 4+60 = 64$

$x = \frac{2 \pm \sqrt{64}}{2(3)}$

$x = \frac{2 \pm 8}{6}$

$\frac{2+8}{6} = \frac{10}{6} = \frac{5}{3}$

$\frac{2-8}{6} = \frac{-6}{6} = -1$

2) $10x^2 = -9$

$10x^2+9=0$

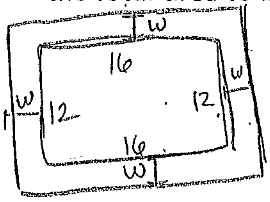
$b^2-4ac = 0^2-4(10)(9) = -360$

$x = \frac{0 \pm \sqrt{-360}}{2(10)}$

No solution

Solve the following quadratic word problems ALGEBRAICALLY.

3) A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?



w = width

$A = L \cdot W$

$285 = (16+2w)(12+2w)$

$285 = 192 + 32w + 24w + 4w^2$

$285 = 192 + 56w + 4w^2$

$\to 4w^2 + 56w - 93 = 0$

$(2w+31)(2w-3) = 0$

~~$w = \frac{-31}{2}$~~

$w = \frac{3}{2}$ or 1.5 meters

4) The sum of two positive numbers is 14 and their product is 32. What are the numbers?

$x + y = 14, \quad y = 14 - x$

$xy = 32$

$x(14-x) = 32$

$14x - x^2 = 32$

$0 = x^2 - 14x + 32$

$b^2 - 4ac = (-14)^2 - 4(1)(32) = 68$

$\to x = \frac{14 \pm \sqrt{68}}{2} = \frac{14 \pm 2\sqrt{17}}{2}$ $7+\sqrt{17}$

$\to y = 14 - (7+\sqrt{17}) = 7 - \sqrt{17}$

5) An object is launched straight up into the air with an initial velocity of 64 ft/sec. It is launched from a height of 6 feet off the ground. Its height, H, in feet, at t seconds is given by the equation $H = -16t^2 + 64t + 6$. Find all times t that the object is at height of 54 feet off the ground.

$54 = -16t^2 + 64t + 6$

$0 = -16t^2 + 64t - 48$

$0 = -16(t^2 - 4t + 3)$

$0 = -16(t-3)(t-1)$

$t = 3$ sec
$t = 1$ sec

6) A clown in a circus is launched at 12.7 meters per second (m/s) from a 39.2-meter tall platform. A function for the clown's height at time t seconds after launch is $S(t) = -4.9t^2 + 12.7t + 39.2$ where s is in meters. When does the clown strike the ground?

$0 = -4.9t^2 + 12.7t + 39.2$

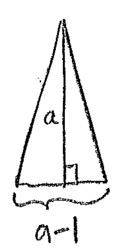
$b^2 - 4ac = (12.7)^2 - 4(-4.9)(39.2) = 929.61$

$t = \frac{-12.7 \pm \sqrt{929.61}}{2(-4.9)}$

~~$t \approx -1.82$ sec~~

$t \approx 4.41$ sec

7) The base of a triangle is 1 centimeter shorter than the altitude. If the area is 15 cm², what is the length of the altitude?



a = altitude

$A = \frac{1}{2}bh$

$15 = \frac{1}{2}(a-1) \cdot a$

$15 = \frac{1}{2}a^2 - \frac{1}{2}a$

$30 = a^2 - a$

$\to 0 = a^2 - a - 30$

$0 = (a-6)(a+5)$

$a = 6$ cm

~~$a = -5$~~

Solve each equation using factoring.

8) $7x^2 - 3 = -4x$

$7x^2+4x-3=0$

$(7x-3)(x+1)=0$

$x=\frac{3}{7}$ $x=-1$

9) $4x^2 - 13x - 31 = 4$

$4x^2-13x-35=0$

$(4x+7)(x-5)=0$

$x=\frac{-7}{4}$ $x=5$

10) $-10 = -13x - 5x^2 - 4$

$0=-5x^2-13x+6$

$0=-(5x^2-13x-6)$

$0=-(5x+2)(x-3)$

$x=-\frac{2}{5}$ $x=3$

11) $16x^3 - 12x^2 - 40x = 0$

$4x(4x^2-3x-10)=0$

$4x(4x+5)(x-2)=0$

$x=0$ $x=\frac{-5}{4}$ $x=2$

Solve the following quadratic inequalities ALGEBRICALLY. State the solution set in interval notation. Draw a number line and shade it accordingly. If required, state the solution in DECIMAL FORM, 2 DECIMAL PLACES.

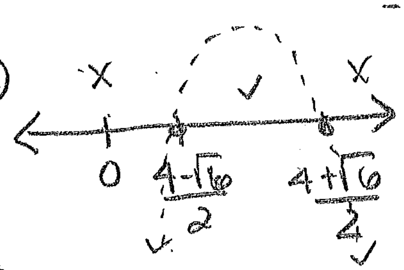
12) $0 < -2x^2 + 8x - 5$ $a=-2$ $b=8$ $c=-5$

① $= \frac{-(8) \pm \sqrt{(8)^2-4(-2)(-5)}}{2(-2)}$

$= \frac{-8 \pm \sqrt{64-40}}{-4} = \frac{-8 \pm \sqrt{24}}{-4}$

$= \frac{8 \pm 2\sqrt{6}}{4} = \frac{4 \pm \sqrt{6}}{2}$

$\approx .78, 3.24$

② 

$(.78, 3.24)$

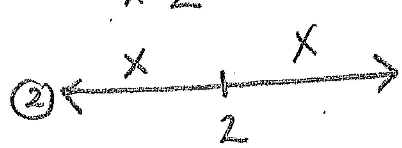
③ Test $x=2$

$0 < -2(2)^2+8(2)-5$ $-8+16-5>0$ ✓

13) $x^2 - 4x + 4 \leq 0$

① $(x-2)(x-2)=0$

$x=2$

② 

no solution

test $x=1$

$(1)^2-4(1)+4 \leq 0$

$1-4+4 \leq 0$

$1 \leq 0$

no

test $x=3$

$(3)^2-3(1)+4 \leq 0$

$9-3+4 \leq 0$

$10 \leq 0$

no

PART VI: QUADRATIC FUNCTIONS $\to y=a(x-h)^2+k$

Write the quadratic equation in vertex form given the following information.

1) Vertex at (-2, 10) & passes through (-6, -12)

$y=a(x+2)^2+10$

$-12=a(-6+2)^2+10$

$-12=16a+10$

$-22=16a$

$-\frac{11}{8}=a$

$\boxed{y=-\frac{11}{8}(x+2)^2+10}$

2) Minimum at (-1, -10) & y-intercept at (0, -9)

$y=a(x+1)^2-10$

$-9=a(0+1)^2-10$

$-9=a-10$

$1=a$

$\boxed{y=(x+1)^2-10}$

Write the following equations in vertex form by completing the square. Then state the vertex and whether it is a minimum or maximum.

3) $x^2+5x+18=y$

$x^2+5x+\underline{6.25}+18=y+\underline{6.25}$

$(x+2.5)^2+11.75=y$

vertex is a minimum at (-2.5, 11.75)

4) $x^2-x=-7$

$x^2-x+\underline{0.25}=-7+\underline{0.25}$

$(x-0.5)^2+6.75=y$

vertex is a minimum at (0.5, 6.75)

5) $f(x)=5x^2-20x-11$

$f(x)=5(x^2-4x+\underline{4})-11$

$f(x)=5(x-2)^2-11$

vertex is a minimum at (2, -11)

6) $-4x^2-56x+19=10+y$

$-4(x^2+14x+\underline{49})+9=y+\underline{-196}$

$-4(x+7)^2+205=y$

vertex is a maximum at (-7, 205)

Determine the type of model that best fits the data (quadratic, linear or exponential) and calculate the average rate of change of the function on the given interval.

7)

x	$f(x)$
−2	1/4
−1	1/2
0	1
1	2
2	4

(×2, ×2, ×2, ×2)

Interval: $[-2,1]$

Exponential

AROC: $(-2,\frac{1}{4})$ $(1,2)$

$\frac{2-\frac{1}{4}}{1--2}=\frac{\frac{7}{4}}{3}=\boxed{\frac{7}{12}}$

8) (-3, -2), (-2, -3), (-1, -1), (0, 4), (1, 12)

(−1, +2, +5, +8)

Interval: $[-3,0]$

Quadratic

AROC: (-3, -2) (0, 4)

$\frac{4--2}{0--3}=\frac{6}{3}=\boxed{2}$

Find the average rate of change of the given function on the given interval. State the type of rate of change that exists and sketch a graph of the parabola with the secant line that represents the average rate of change on the interval.

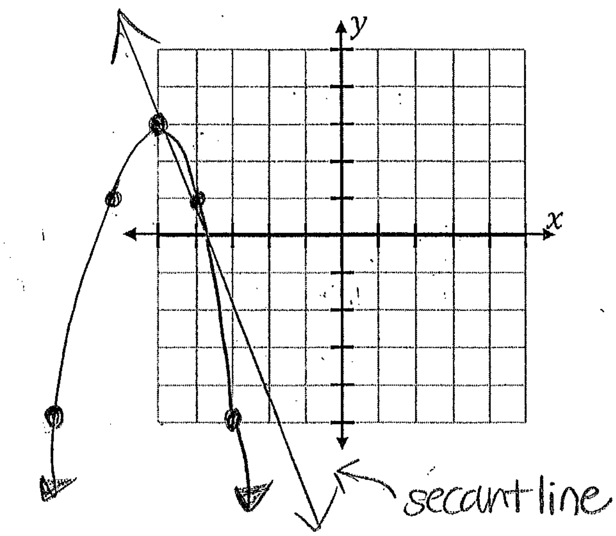
9) $f(x) = -2(x+5)^2 + 3$
Interval: $[-5,-4]$

vertex at $(-5,3)$

AROC on $[-5,-4]$:

$(-5,3)(-4,1)$

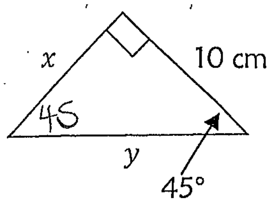
$\frac{1-3}{-4+5} = \frac{-2}{1} = \boxed{-2}$



PART VII: RIGHT TRIANGLE TRIGONOMETRY

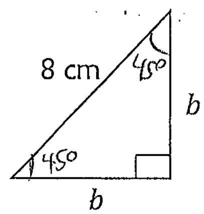
Find the value of all variables below using special right triangle ratios. Do not leave radicals in the denominators of your fractions.

1)



$x = 10$ cm
$y = 10\sqrt{2}$ cm

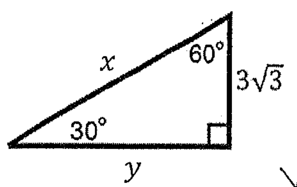
2)



$b = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$

$b = 4\sqrt{2}$ cm

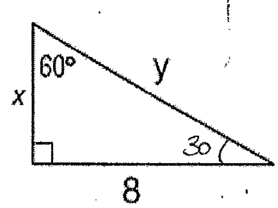
3)



$x = 2 \cdot 3\sqrt{3} = \boxed{6\sqrt{3}}$

$y = 3\sqrt{3} \cdot \sqrt{3} = \boxed{9}$

4)

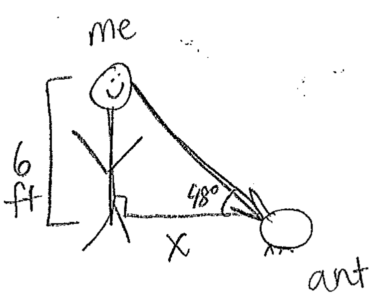


$x = \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{8\sqrt{3}}{3}}$

$y = \boxed{\frac{16\sqrt{3}}{3}}$

Use a trigonometric ratio to solve the following problems.

5) An ant is looking up at you with an angle of elevation of 48°. You are 6 feet tall. How far is the ant from your foot? Round to 2 decimal places.

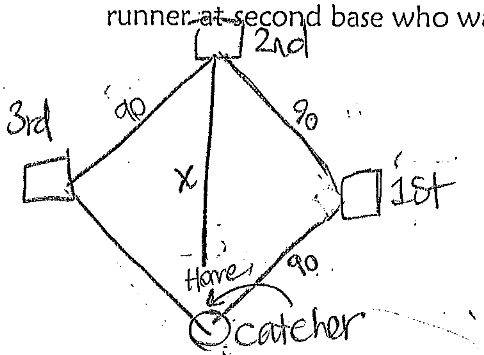


$\tan 48° = \frac{6}{x}$

$x \cdot \tan 48° = 6$

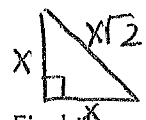
$x = \frac{6}{\tan 48°} \approx \boxed{5.40 \text{ ft}}$

6) A baseball diamond is in the shape of a square with each side being 90 feet. If the catcher throws out a runner at second base who was trying to steal, how far does he need to throw the ball?

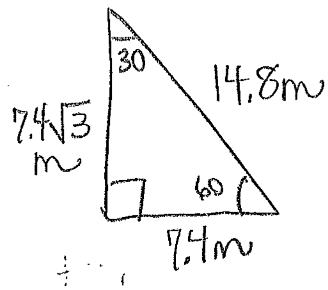


$90\sqrt{2} = 127.28$ ft

← special right triangle 45-45-90

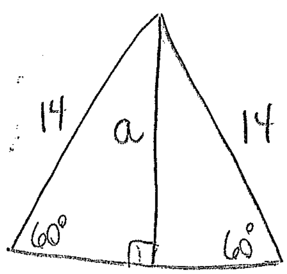


7) The shorter leg of a 30° – 60° – 90° triangle is 7.4 meters long. Find the perimeter.



Perimeter ≈ 25.02 meters

8) Find the altitude of an equilateral triangle if each side of the triangle has a length of 14 meters.

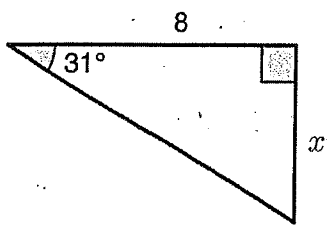


$\sin 60° = \frac{a}{14}$

$14 \cdot \sin 60° = a$

$12.12 \approx a$ meters

9) Solve for x.



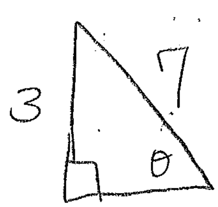
$\tan 31° = \frac{x}{8}$

$8 \cdot \tan 31° = x$

$4.81 \approx x$

Find the value of the missing angle.

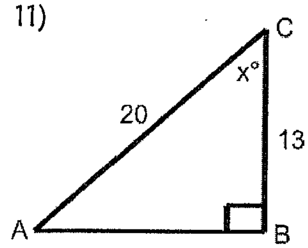
10) If $\sin \theta = \frac{3}{7}$, find θ.



$\theta = \sin^{-1}\left(\frac{3}{7}\right)$

$\theta \approx 25.38°$

11)



$\cos x° = \frac{13}{20}$

$x = \cos^{-1}\left(\frac{13}{20}\right)$

$x \approx 49.46°$

PART VIII: SOLVING SYSTEMS OF EQUATIONS

Solve the system of equations algebraically.

1) $25x - 5y = 30$
$-35x + 7y = -7$

$\rightarrow 5x - y = 6$

$y = 5x - 6$

$-35x + 7(5x-6) = -7$

$-35x + 35x - 42 = -7$

$-42 \neq -7$

No solution

2) $8x - y + 3z = -38$
$2x + 5y - 4z = 32$
$x - y + z = -9$

$8x - y + 3z = -38$
$-x + y - z = 9$
$7x + 2z = -29$

$2x + 5y - 4z = 32$
$5x - 5y + 5z = -45$
$7x + z = -13$

$7x + z = -13$
$-7x - 2z = 29$
$-z = 16$
$z = -16$

$7x - 16 = -13$
$7x = 3$
$x = \frac{3}{7}$

$\frac{3}{7} - y - 16 = -9$
$-y - \frac{109}{7} = -9$
$-y = \frac{46}{7}$
$y = -\frac{46}{7}$

$\left(\frac{3}{7}, -\frac{46}{7}, -16\right)$

3) $5x - 2y = 14$
$-3x + y = -7$

$y = 3x - 7$

$5x - 2(3x - 7) = 14$

$5x - 6x + 14 = 14$

$-x = 0$

$x = 0$

$5(0) - 2y = 14$

$-2y = 14$

$y = -7$

$(0, -7)$

4) $-4x - 2y - z = 5$
$2x - z = 8$
$y - 2z = -4$

$\rightarrow 2x = z + 8, x = \frac{z}{2} + 4$

$\rightarrow y = 2z - 4$

$-4\left(\frac{z}{2} + 4\right) - 2(2z - 4) - z = 5$

$-2z - 16 - 4z + 8 - z = 5$

$-7z - 8 = 5$

$-7z = 13$

$z = -\frac{13}{7}$

$y = 2\left(-\frac{13}{7}\right) - 4$
$y = -\frac{26}{7} - 4$
$y = -\frac{54}{7}$

$2x + \frac{13}{7} = 8$
$2x = \frac{43}{7}$
$x = \frac{43}{14}$

$\left(\frac{43}{14}, -\frac{54}{7}, -\frac{13}{7}\right)$

PART IX: PIECEWISE FUNCTIONS

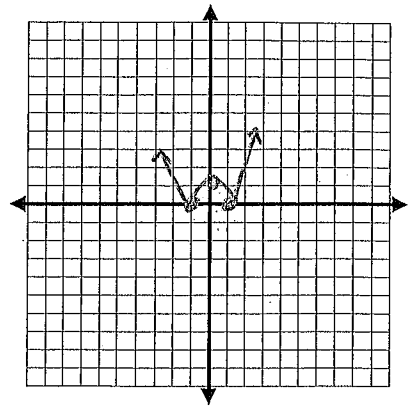
Graph the following functions. Then write the absolute value function as a piecewise function.

1) $f(x) = |x^2 - 1|$ → vertex: (0, −1) → so absolute value (0, 1)

$x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$

Piecewise function:

$$f(x) = \begin{cases} x^2 - 1; & -\infty < x \le -1 \\ -(x^2 - 1); & -1 < x < 1 \\ x^2 - 1; & 1 \le x < \infty \end{cases}$$

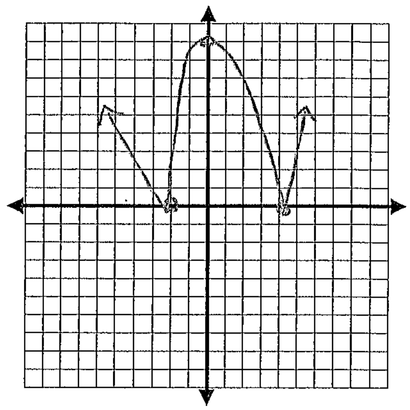


2) $f(x) = |(x-1)^2 - 9|$ → vertex: (1, −9) → (1, 9)

Piecewise function:

$(x-1)^2 - 9 = 0$
$(x-1)^2 = 9$
$x - 1 = \pm 3$
$x = 1 \pm 3$
$1 - 3 = -2$, $1 + 3 = 4$

$$f(x) = \begin{cases} (x-1)^2 - 9; & -\infty < x \le -2 \\ -(x-1)^2 + 9; & -2 < x < 4 \\ (x-1)^2 - 9; & 4 \le x < \infty \end{cases}$$



Graph the following functions. Then evaluate them at requested values.

3) $h(x) = \begin{cases} -e^x + 7, -\infty < x < 0 \\ 3, 0 < x < 3 \\ |2x + 2|, 3 \le x < 4 \end{cases}$

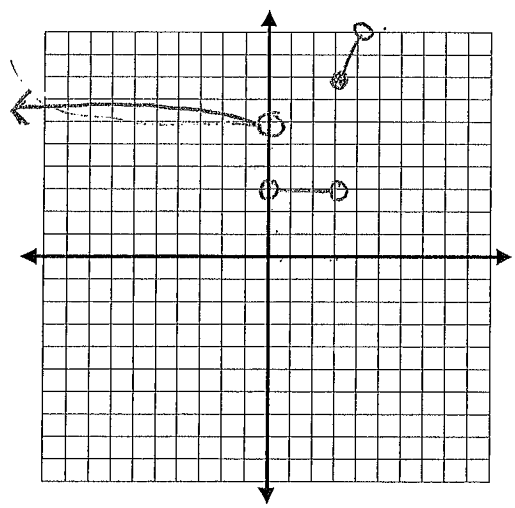
x	y
0	6
−3	6.95

x	y
3	8
4	10

a. $h(-3) =$ 6.95

b. $h(0) =$ undefined

c. $h(2) =$ 3



4) $k(x) = \begin{cases} 3 - x, x < -2 \\ \frac{4}{x-1}, -2 < x < 3 \\ (x-4)^3, 3 \le x < 6 \end{cases}$

x	y
−2	5
−3	6

x	y
−2	−1.33
3	2
−1	−2
1	undefined

VA: x = 1
HA: y = 0

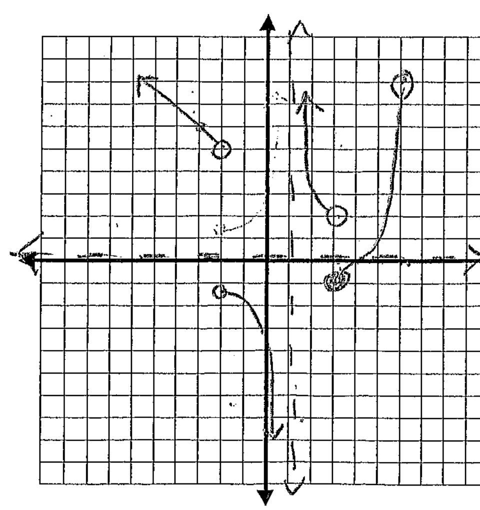
x	y
3	−1
6	8

inflection pt: (4, 0)

a. $k(-1) =$ −2

b. $k(1) =$ undefined (vertical asym)

c. $k(5) =$ 1



PART X: EXPONENTIAL FUNCTIONS

1) The population of a species of ant grows at a rate of 13% every six months. If there are initially 85 ants in the colony, how many would be present after 3.5 years?

→ 42 months

$$P(t) = 85(1.13)^{42/6}$$

$\approx$ 199 or 200 ants

2) The half-life of a radioactive isotope is 10 years. Initially, there are 500 grams of the isotope in the laboratory. How much of the isotope would remain after 33 years?

$$y = 500\left(\frac{1}{2}\right)^{33/10} \approx 50.766 \text{ grams}$$

3) The value of a car depreciates by 17% a year. If a car is initially purchased for $28,000, what would be the worth of the car after 5 years?

$$V(t) = 28000(1-.17)^5 \approx \$11,029.31$$

4) Andi invests $7,500 in a savings account that pays 2.55% interest compounded monthly. How much money will be in the account after 20 years?

→ .0255

$$A = 7500\left(1+\frac{.0255}{12}\right)^{12(20)} \approx \$12,482.93$$

5) The population of Hanover was 38,721 in the year 1990. The population has grown consistently at a rate of 1.6% each year. What was the population in the year 2010 (the last census) and what would be the predicted population for the year 2014?

$$P = 38721(1+.016)^t$$

2010 ($t=20$) → 53188

2014 ($t=2014$) → 56675

6) The population of Hancock County, GA has declined steadily at a rate of 0.9% each year. In the year 1990, the population was approximately 8,750. What was the population in the year 2010, and would be the expected population for the year 2016?

$$P = 8750(1-.009)^t$$

2010 ($t=20$) → 7302

2016 ($t=26$) → 6917

PART XI: RULES OF EXPONENTS

Use properties of exponents to simplify the expressions completely.

1) $b^4 \cdot b^2$ $\quad b^{4+2} = \boxed{b^6}$

2) $x^{-3} \cdot x^5$ $\quad x^{-3+5} = \boxed{x^2}$

3) $\frac{m^2}{m^6}$ $\quad m^{2-6} = m^{-4} = \boxed{\frac{1}{m^4}}$

4) $\left(\frac{x}{x^2 y}\right)^2$ $\quad \frac{x^2}{x^4 y^2} = \left(\frac{x^2}{x^4}\right)\left(\frac{1}{y^2}\right) = \boxed{\frac{1}{x^2 y^2}}$

5) $(5y)^2$ $\quad 5^2 y^2 = \boxed{25y^2}$

6) $\left(\frac{z^9}{3z^5}\right)^{-1}$ $\quad \left(\frac{3z^5}{z^9}\right) = \boxed{\frac{3}{z^4}}$

7) $\left(\frac{n}{4m^2 n}\right)^{-2}$ $\quad (4m^2)^2 = \boxed{16m^4}$

8) $\frac{2^{-4} 2^2}{(2^2)^{-4}}$ $\quad \frac{2^{-2}}{2^{-8}} = 2^{-2-{-8}} = 2^6 = \boxed{64}$

9) $\frac{\left(2(2^2)^{-3}\right)^2}{2^{-5}}$ $\quad \frac{(2(2^{-6}))^2}{2^{-5}} = \frac{(2^{-5})^2}{2^{-5}} = 2^{-5} = \frac{1}{2^5} = \boxed{\frac{1}{32}}$

10) $\frac{(-x^{-1} y^2)^{-1}}{x^{-2} y^3 (x^0 y^{-2})^3}$ $\quad = \frac{-1 \cdot x^1 \cdot y^{-2}}{x^{-2} \cdot y^3 (x^0) y^{-6}} = \frac{-xy^{-2}}{x^{-2} y^{-3}} = \boxed{-x^3 y}$

11) $\frac{x^{-3} y^3}{-x^4 y^5 (-y^{-3})^2}$ $\quad = \frac{x^{-3} \cdot y^3}{-1 \cdot x^4 \cdot y^5 \cdot (-1)^2 \cdot (y^{-3})^2} = \frac{x^{-3} y^{-3}}{-x^4 y^5 y^{-6}} = \boxed{\frac{-1}{x^7 y^2}}$

PART XII: SIMPLIFYING RADICALS

Simplify the following expressions as much as possible.

1) $\sqrt{75x^7y^5}$ $= \sqrt{3\cdot 25\cdot x^6\cdot x\cdot y^4\cdot y}$

$= 5x^3y^2\sqrt{3xy}$

2) $\sqrt{27a^{11}b^7}$ $= \sqrt{9\cdot 3\cdot a^{10}\cdot a\cdot b^6\cdot b}$

$= 3a^5b^3\sqrt{3ab}$

3) $\sqrt{32a^7b^4}$

$= \sqrt{16\cdot 2\cdot a^6\cdot a\cdot b^4} = 4a^3b^2\sqrt{2a}$

4) $\frac{4}{\sqrt{8}}$ $= \frac{2^2}{\sqrt{2^3}} = \frac{2^2}{2^{3/2}} = 2^{2-3/2} = 2^{1/2}$

$= \sqrt{2}$

5) $\frac{5}{\sqrt{125}}$ $= \frac{5}{\sqrt{5^3}} = \frac{5}{5^{3/2}} = 5^{1-\frac{3}{2}} = 5^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{5}}$

6) $-\frac{4}{3\sqrt{2}}$ $= -\frac{2^2}{3\cdot 2^{1/2}} = \frac{-1}{3}\cdot 2^{(2-1/2)}$

$= -\frac{1}{3}\cdot 2^{3/2} = \frac{-2\sqrt{2}}{3}$

7) $4\sqrt{5} + \sqrt{125} + \sqrt{45}$

$4\sqrt{5} + \sqrt{25\cdot 5} + \sqrt{9\cdot 5}$

$4\sqrt{5} + 5\sqrt{5} + 3\sqrt{5} = 12\sqrt{5}$

8) $\sqrt{48} + 10 - \sqrt{100} - \sqrt{98}$

$\sqrt{16\cdot 3} + 10 - 10 - \sqrt{49\cdot 2}$

$4\sqrt{3} - 7\sqrt{2}$

9) $2x\sqrt{3x^2} + 5 - 7\sqrt{3x^2} - 2$

$2x^2\sqrt{3} + 5 - 7x\sqrt{3} - 2$

$= 2x^2\sqrt{3} - 7x\sqrt{3} + 3$

10) $3x\sqrt{18} - 3\sqrt{98x^2} + 2x\sqrt{12x^2}$

$3x\sqrt{9\cdot 2} - 3\sqrt{49\cdot 2\cdot x^2} + 2x\sqrt{4\cdot 3\cdot x^2}$

$9x\sqrt{2} - 21x\sqrt{2} + 4x^2\sqrt{3}$

$= -12x\sqrt{2} + 4x^2\sqrt{3}$

PART XIII: COMPLEX NUMBERS

Simplify the following expressions as much as possible.

1) $(3i^{31} - 3)(4 + i^{57})$

$12i^{31} + 3i^{88} - 12 - 3i^{57}$

$12(i^2)^{15}i + 3(i^2)^{44} - 12 - 3(i^2)^{28}\cdot i$

$12(-1)^{15}\cdot i + 3(-1)^{44} - 12 - 3(-1)^{28}\cdot i$

$= -12i + 3 - 12 - 3i$

$= -15i - 9$

2) $(i\sqrt{2} - 6) - (3i\sqrt{32} + 15) + (-2i\sqrt{18} - 11)$

$i\sqrt{2} - 6 - 3i\sqrt{16\cdot 2} - 15 - 2i\sqrt{9\cdot 2} - 11$

$= i\sqrt{2} - 6 - 12i\sqrt{2} - 15 - 6i\sqrt{2} - 11$

$= -17i\sqrt{2} - 32$

3) $i\sqrt{3}(-2 - i\sqrt{6})$

$-2i\sqrt{3} - i^2\sqrt{3\cdot 3\cdot 2}$

$= -2i\sqrt{3} + 3\sqrt{2}$

4) $\frac{3-4i}{3i}\cdot\frac{3i}{3i}$

$= \frac{9i - 12i^2}{9i^2} = \frac{9i + 12}{-9}$

$= -i - \frac{4}{3}$